APPLICATION NO.   09/826,118

TITLE OF INVENTION: Wavelet Multi-Resolution Waveforms

INVENTOR: Urbain A. von der Embse

Clean version of how the CLAIMS will read.

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CLAIMS

WHAT IS CLAIMED IS:

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Claim **1**. (currently amended) An iterative eigenvalue
least-squares LS method for designing digital mother Wavelets at
baseband for multi-resolution waveforms and filters, said method
comprising steps:

15 power spectral density PSD representative requirements for said

mother Wavelet ψ frequency ω response $\psi_\omega(\omega)$ in a multi-
channel filter bank, specify

a) passband frequency range for waveform transmission,

b) stopband spacing between adjacent filters,

20 c) bounds on ripple over said passband,

d) stopband filter attenuation,

e) rolloff with frequency outside stopband,

f) quadrature mirror filters QMF require the sum of said

PSD's for contiguous filter responses to be flat over

25 deadband which is said stopband,

g) symbol-to-symbol interference ISI,

h) adjacent channel interference ACI,

said LS error metrics to measure said requirements (a)-(h) are

derived as functions of said Wavelet $\psi(n)$ assuming

30 i) T and 1/T are sample interval and sample rate equal to

Nyquist sample rate,

j) ψ is real and symmetric about n=0,

k) n= 0, +/-1, . . ., +/-ML/2 digital index over said ψ,

l) M is interval between contiguous said ψ,

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m) 1/MT is said ψ symbol rate and channel-to-channel
separation,

n) L is length of said ψ in units of said M,

said multiple-resolution properties require said LS metrics
to be constructed as functions of said Wavelet Fourier
harmonics $\psi_k(k)$ with $k=0, +/-1, \ldots, +/-(N_k-1)$ and
$N_k \geq L$ is a design parameter,

it is sufficient to use positive $n=0,1,\ldots, ML/2$ and $k=0,1,\ldots$
$\ldots, N_k-1$ since said $\psi(n)$ and $\psi_k(k)$ are real and symmetric,

$ML/2+1$ x N_k matrix bw wherein "x" reads "by" maps $\psi_k(k)$ into
$\psi(n)$ to within a scale factor by equations

$$\psi(n) = \sum_k bw(n+1,k+1)\psi_k(k) \quad \text{for } n\geq 0, \ k\geq 0,$$

$$bw(n+1,k+1) = 1 \qquad \text{for } n=0,$$

$$= 2\cos(2\pi nk/ML) \quad \text{otherwise,}$$

$$= \text{row } n+1, \text{ column } k+1 \text{ element of bw,}$$

said $\psi(n)$ and $\psi_k(k)$ are converted to column vectors by equations

$h = ML/2+1$ x 1 column vector of $\psi(n)$ with elements

$$h(1) = \psi(n=0),$$

$$h(n+1) = 2\psi(n) \quad \text{for } n=1,2,\ldots ML/2,$$

$h_k = N_k$ x 1 column vector of $\psi_k(k)$ with elements

$$h_k(1) = \psi_k(k=0),$$

$$h_k(k+1) = 2\psi_k(k) \quad \text{for } k=1,2,\ldots,N_k-1,$$

said bw maps h_k into h to within a scale factor by matrix equation

$$h = bw\ h_k,$$

LS error metrics for band = passband, stopband, and QMF deadband
requirements are derived as quadratic forms in said h,

said LS error metrics are converted by said bw mapping into
quadratic forms in said h_k equal to $J(band)=h_k'R\ h_k$ wherein
said h_k' is the transpose of h_k and said R is a real square
symmetric matrix of LS errors in meeting said requirements.

LS ISI,ACI error metrics $J(ISI),J(ACI)$ are derived as non-linear

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quadratic forms in h and converted by said bw matrix to the non-linear quadratic form in h_k equal to $J(ISI) = \delta E' \delta E$,

$J(ACI) = 2\delta E' \delta E$ wherein $\delta E = AHh_k$ is a column vector and matrix "A" in the matrix product AH is a function of said h

5   hereby introducing said non-linearity, and said AH differ for ISI and ACI error metrics,

LS cost function J is the weighted sum of said LS error metrics

$$J = \sum w(LS\ metric)\ J(LS\ metric)$$

with summation over said LS metrics= passband, stopband,

10   QMF deadband, ISI, ACI with normalized weights

$$\sum w(LS\ metric) = 1,$$

said weights are free design parameters,

said iterative eigenvalue LS algorithm at each step finds the optimum eigenvalue and eigenvector which minimize said

15   quadratic form J in h_k for a constant said "A",

said eigenvector is the optimum h_k which minimizes said J and said bw equation derives the corresponding optimum h which minimizes said J,

step 1 in said iterative algorithm finds said optimum eigenvalue,

20   eigenvector, h_k, h of J reduced by deleting said non-linear ISI and ACI LS quadratic error metrics,

said h is used to evaluate said "A" matrices for step 2,

step 2 finds said optimum eigenvalue, eigenvector, h_k, h for minimum J using said "A" from step 1,

25   said h is used to evaluate said "A" for step 3,

steps 3,4, etc. continue until said minimum J converges to a steady value and,

said optimum $\psi_k(k)$ uses said bw to calculate optimum $\psi(n)$ for implementation as said Wavelet FIR digital waveform and

30   filter time response.

Claim **2.** (currently amended) An LS method for designing
digital mother Wavelets at baseband for multi-resolution
waveforms and filters, said method comprising steps:
said PSD waveform representative requirements and assumptions
5   are recited in (a)-(n) in Claim **1**,
said multiple-resolution properties require said LS metrics
        to be constructed as functions of said $\psi_k(k)$,
said LS error metrics for said passband, stopband, and QMF
        deadband requirements are derived as squared vector norm
10   functions of said h and converted by said bw matrix into

$$J(band) = \left\| Bh_k \right\|^2 \text{ wherein } \left\| Bh_k \right\| \text{ is the vector norm of the}$$

        column vector Bh_k and said B is the matrix of LS errors in
        meeting said requirements and wherein said squared vector
        norm is suitable for LS optimization,
15   LS ISI,ACI error metrics J(ISI),J(ACI) are derived as squared

$$\text{vector norm functions equal to } J(ISI = \left\| \delta E \right\|^2, \quad J(ACI) = 2 \left\| \delta E \right\|^2$$

        using said column vectors $\delta E = AHh_k$ in claim **1**,
    LS cost function J is said weighted sum of said LS error metrics
        equal to $J = \sum w(LS \text{ metric}) J(LS \text{ metric})$ defined in claim **1**,
20   an LS gradient search algorithm finds optimum $h_k(k)$ to
        minimize J,
    step 1 of said LS gradient search algorithm uses a Remez-
        exchange algorithm to find said optimum $h_k(k)$ for said J
        reduced to said passband and stopband LS metrics,
25   step 2 uses the estimated $h_k(k)$ from step 1 to initialize said
        gradient search,
    step 3 selects one of several available gradient search
        algorithms, gradient search parameters, and stopping rules,
    step 4 implements said algorithm, parameters, and stopping rule
30   selected in step 3 to derive said optimum $h_k(k)$ to minimize
        J and,
    said optimum h_k uses said bw to calculate optimum $\psi(n)$ for

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implementation as said Wavelet FIR digital waveform and filter time response.

5   Claim **3**. (currently amended) Wherein said mother Wavelet in claims **1,2** generates multi-resolution dilated Wavelets, comprising steps and design:
said Wavelet parameters are
           a) scaling parameter p dilates sampling by factor 2^p
10   equivalent to sub-sampling by factor 2^p,
           b) translation parameter q translates said ψ by qM digital samples,
           c) frequency offset k is set by design,
           d) symbol repetition interval said M remains constant,
15   e) Wavelet length said L in units of said M remains constant,
step 1 uses said design harmonics ψ_k to generate said FIR time response $\psi(n_p)$ at baseband with said bw equation
$$\psi(n_p) = \sum_k bw(n_p+1, k+1) \; \psi_k(k)$$
20   recited in claim **1** with
           $bw = 2 \cos(2\pi \; n_p k/ML)$ for $n_p > 0$
           wherein $n_p = n/2^p$ is n sub-sampled or equivalently dilated by the factor 2^p,
step 2 uses said $\psi(n_p)$ to construct said multi-resolution
25   Wavelet $\psi_{p,q,M,L,k}$ with equation .
           $$\psi_{p,q,M,L,k} = 2^{(-p/2)} \; \psi(n_p - qM) \; \exp(j2\pi \; kn_p/ML)$$
           which is said FIR time response for parameters p,q,M,L,k
           wherein the subset p,M,L are the scale parameters,
design of said multi-resolution Wavelet includes
30   f) said T for n is increased to $T2^p$ for n_p,
           g) said 1/T is reduced to $1/T2^p$,
           h) said ψ symbol rate 1/MT equal to said channel-to-channel separation is reduced to $1/MT2^p$ in Hz and,

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i) said ψ length $(ML+1)T$ in seconds is stretched to
$(ML+1)T2^p$ in seconds.

5   Claim **4**. (currently amended) Wherein said mother Wavelet in
claims **1,2** generates multi-resolution constant sample rate
dilated Wavelets, comprising steps and design:
said Wavelet parameters are
        a) said p dilates said ψ to increase said length from
10          $ML+1$ to M_pL+1 where $M_p=M2^p$ is the dilated interval
            between contiguous ψ's,
        b) said q translates said ψ by qM_p digital samples,
        c) said k is set by design,
        d) said $M_p=M2^p$ is dilated M,
15   e) said L remains constant,
step 1 uses said design harmonics ψ_k to generate said FIR time
        response $\psi(n_p)$ at baseband with said bw equation
            $$\psi(n) = \sum_k bw(n+1,k+1) \; \psi_k(k)$$
        recited in claim **1** with
20          $$bw = 2 \cos(2\pi \; nk/M_pL) \text{ for } n>0,$$
step 2 uses said $\psi(n)$ to construct said multi-resolution Wavelet
        $\psi_{p,q,M,L,k}$ with equation
            $$\psi_{p,q,M,L,k} = 2^{(-p/2)} \; \psi(n-qM_p) \; \exp(j2\pi \; kn/M_pL)$$
        which is said FIR time response for parameters p,q,M,L,k,
25 design of said multi-resolution Wavelet includes
        f) said T remains constant,
        g) said $1/T$ remains constant,
        h) said ψ symbol rate $1/MT$ equal to said channel-to-channel
            separation is reduced to $1/M_pT=1/MT2^p$ in Hz and,
30   i) said ψ length $(ML+1)T$ in seconds is stretched to
            $(ML2^p+1)T$ in seconds.

Claim **5**. (currently amended) Wherein said mother Wavelet in
claims **1,2** generates multi-resolution up-sampled Wavelets,
comprising steps and design:
said Wavelet parameters are

5   a) said p up-samples said digital sampling rate 1/T to 2^p/T,

     b) said q translates said ψ by qM digital samples,

     c) said k is a design parameter,

     d) said M is constant

     e) said L is constant

10   step 1 uses said design harmonics h_f, ψ_k to generate said FIR

        time response $\psi(n_p)$ at baseband with said equation

$$\psi(n_p) = \sum_k bw(n_p+1, k+1) \ \psi_k(k)$$

        recited in claim 1 with

$$bw = 2 \cos(2\pi \ n_p k/ML) \ \text{for} \ n_p > 0$$

15   wherein n_p is n up-sampled by the factor 2^p and defined

        by equations

$$n_p = n\_p + n \ 2^p$$
$$n\_p = 0, 1, 2, \ldots, 2^p - 1$$

        wherein n_p is the index over the additional samples added

20   to each sample n by said up-sampling,

   step 2 uses said $\psi(n_p)$ to construct said multi-resolution Wavelet

        $\psi_{p,q,M,L,k}$ with equation

$$\psi_{p,q,M,L,k} = 2^{(-p/2)} \ \psi(n_p - qM) \ \exp(j2\pi \ kn_p/ML)$$

        which is said FIR time response for parameters p,q,M,L,k,

25   design of said multi-resolution Wavelet includes

       f) said T is decreased to   T/2^p,

       g) said 1/T is increased to 2^p/T,

       h) said ψ symbol rate 1/MT equal to said channel-to-channel

         separation is increased to   2^p/MT in Hz and,

30   i) said ψ length (ML+1)T in seconds is reduced to

         (ML +1)T/2^p in seconds.

Claim **6** (currently amended) Wherein said multi-resolution
Wavelets in Claims **1-5** have properties comprising:

said scale parameters p,M,L and said design parameter 1/T specify
   said multi- resolution Wavelets at baseband and said q,k
5   specifify time,frequency translations from baseband,

said design harmonics $\psi_k(k)$ of mother Wavelet are said design
   coordinates for multi-resolution Wavelets,

said design harmonics $\psi_k(k)$ use said bw matrix to generate said
   multi-resolution Wavelet baseband time response $\psi(n)$ for
10   dilation, dilation of Wavelet length, and up-sampling as
   recited in Claims **3-5** and which is translated in time and
   frequency to said multi-resolution Wavelet $\psi_{p,q,M,L,k}$,

said design harmonics $\psi_k(k)$ are few in number compared to said
   $\psi(n)$,

15 said ψ is designed to support a bandwidth-time product $B_fT=1+\alpha$
   with no zero excess bandwidth $\alpha=0$,

said multi-resolution Wavelets are designed to behave like an
   accordion in that at different scales said Wavelets are
   stretched and compressed versions of the mother Wavelet with
20   appropriate time and frequency translation,

said optimization techniques in claims **1,2** assume said $\psi(n)$
   symmetric about n=0 and are applicable to other arrangements
   of $\psi(n)$ with self-evident modifications,

optimization algorithms for finding said optimum set of $\psi_k(k)$
25   use linear LS waveform and filter design methods recited in
   claims **1,2** and also use other methods and,

said linear waveform and filter LS design methods can be modified
   to design waveforms for applications including bandwidth
   efficient modulation BEM and synthetic aperture radar SAR.